

**Lecture Notes on**

**MONEY, BANKING,  
AND FINANCIAL MARKETS**

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# Chapter 16: Determinants of the Money Supply

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By working through the details of Mishkin's Chapter 15, we saw how the Federal Reserve can change the level of deposits by changing the level of reserves supplied to the banking system, relying on the formula

$$\Delta D = \frac{1}{r} \times \Delta R.$$

But to derive this formula, we had to make two simplifying assumptions:

Banks never hold excess reserves.

Individuals and non-bank corporations never hold currency.

By working through Chapter 15, we also saw how the Fed can control the monetary base much more precisely than it can control reserves.

Thus, by working through Chapter 16, our goals will be two-fold:

First, we will extend our simple model of multiple deposit creating by dispensing with the two simplifying assumptions. Now, we will allow banks to hold excess reserves and all individuals and non-bank corporations to hold currency as well as deposits.

Second, we will extend our simple model so that, from the Fed's perspective, it focuses on changes in the monetary base rather than changes in reserves. At the same time, we will extend the model so that it explains movements in the money supply as a whole rather than just movements in deposits.

The link between the monetary base, which the Fed controls, and the money supply as a whole involves the money multiplier. Thus, we'll begin our analysis by defining this money multiplier.

Then we will identify a number of factors that determine the money multiplier.

One of these will be the required reserve ratio, an object that has already figured importantly into our analysis of multiple deposit creation.

A second factor that will emerge in our extended analysis will be the currency ratio, which describes depositors' decisions regarding their holdings of currency.

And a third factor will be the excess reserves ratio, which describes banks' decisions regarding their holdings of excess reserves.

Finally, we'll conclude our analysis of Chapter 16—and of this fifth part of the course as a whole—with a brief overview of the money supply process.

## 1 The Money Multiplier

Since the Fed can control the monetary base much more precisely than it can control reserves, it makes sense to model the money supply process by linking the money supply to the monetary base.

Let

$C$  = currency in circulation

$D$  = checkable deposits (demand deposits + NOW accounts)

$M = C + D$  = money supply (M1, but ignoring traveler's checks)

$MB$  = monetary base = high-powered money.

The money multiplier links the money supply  $M$  to the monetary base  $MB$  via

$$M = m \times MB$$

where

$m$  = money multiplier.

Our subsequent analysis will reveal that:

$m > 1$ , so that a \$1 increase in  $MB$  leads to an increase in  $M$  of more than \$1. For this reason, the monetary base is often called high-powered money.

$m$  will depend on depositors' decisions about holdings of currency and banks' decisions about holdings of excess reserves.

## 2 Deriving the Money Multiplier

Recall that in our simple model of multiple deposit creation, we assumed that banks never hold excess reserves and that depositors never hold currency.

Now, let's introduce some additional notation that will allow us to relax these assumptions.

Let

$C$  = currency in circulation

$D$  = deposits

$ER$  = excess reserves,

and define

$c = C/D$  = currency ratio

= measures how much currency the non-bank public holds

$e = ER/D$  = excess reserve ratio

= measures how much excess reserves banks hold

Next, let

$R$  = total reserves

$RR$  = required reserves

$ER$  = excess reserves,

so that

$$R = RR + ER. \tag{1}$$

As in our previous analysis, let

$r$  = required reserve ratio

so that required reserves can be calculated as

$$RR = r \times D. \tag{2}$$

Now substitute equation (2) into equation (1):

$$R = r \times D + ER. \quad (3)$$

Finally, use equation (3) to write

$$MB = R + C = r \times D + ER + C. \quad (4)$$

Before going any further with our derivations, let's ask: what are the implications of equation (4)?

Consider 3 examples:

Example 1: Let  $ER = 0$  and  $C = 0$ . Then (4) becomes

$$MB = r \times D + ER + C \quad (4)$$

$$MB = r \times D$$

$$D = \frac{1}{r} \times MB.$$

Since  $r < 1$ ,  $1/r > 1$ . Hence, this example reveals that a \$1 increase in  $MB$  that flows into deposits gets multiplied: it leads to a increase in deposits of more than \$1, due to the process of multiple deposit creation.

Note that in developing our simple model of deposit creation, we were assuming that  $ER = 0$  and  $C = 0$ , so that banks never hold excess reserves and depositors never hold currency. So it's not surprising that this first example leads us to exactly the same conclusions as before. But with our more general model we can consider additional examples that depart from our earlier assumptions.

Example 2: Let  $D = 0$  and  $C = 0$ . Then (4) becomes

$$MB = r \times D + ER + C \quad (4)$$

$$ER = MB.$$

Thus, a \$1 increase in  $MB$  that is held as excess reserves does not get multiplied: excess reserves do not participate in the process of multiple deposit creation, since they are held by banks instead of being lent out.

Example 3: Let  $D = 0$  and  $ER = 0$ . Then (4) becomes

$$MB = r \times D + ER + C \quad (4)$$

$$C = MB.$$

Thus, a \$1 increase in  $MB$  that is held as currency does not get multiplied: currency does not participate in the process of multiple deposit creation, since it is not deposited.

Conclusion: While an increase in  $MB$  that flows into deposits is multiplied, excess reserves and currency do not participate in the process of multiple deposit creation.

To complete our derivation of the money multiplier, let's rewrite equation (4) in terms of the currency and excess reserve ratios:

$$c = C/D \text{ or } C = c \times D$$

$$e = ER/D \text{ or } ER = e \times D.$$

Return to equation (4):

$$MB = r \times D + ER + C \tag{4}$$

$$MB = r \times D + e \times D + c \times D$$

$$MB = (r + e + c) \times D$$

$$D = \frac{1}{r + e + c} \times MB. \tag{5}$$

Now use our definition of the money supply  $M$ :

$$M = C + D$$

$$M = c \times D + D$$

$$M = (1 + c) \times D. \tag{6}$$

Substitute equation (5) into equation (6):

$$M = \frac{1 + c}{r + e + c} \times MB$$

or, more simply,

$$M = m \times D,$$

where now we have a specific expression for the money multiplier  $m$ :

$$m = \frac{1 + c}{r + e + c}.$$

### 3 Factors that Determine the Money Multiplier

Since, according to our formula,

$$m = \frac{1 + c}{r + e + c},$$

it appears that the money multiplier  $m$  is determined by three factors:

1. The required reserve ratio  $r$ .
2. The currency ratio  $c$ .
3. The excess reserve ratio  $e$ .

Let's consider a specific example, to see exactly how this formula works:

$$r = 10\% = 0.10$$

$$C = \$400 \text{ billion}$$

$$D = \$800 \text{ billion}$$

$$ER = \$0.8 \text{ billion} = \$800 \text{ million}$$

$$M = C + D = \$1200 \text{ billion}$$

Thus, in this example,

$$c = C/D = \frac{\$400 \text{ billion}}{\$800 \text{ billion}} = 0.5,$$

so that depositors hold 50% of their funds as currency, and

$$e = E/D = \frac{\$0.8 \text{ billion}}{\$800 \text{ billion}} = 0.001,$$

so that banks hold 0.1% of their deposits as excess reserves.

Our formula for the money multiplier implies:

$$m = \frac{1 + c}{r + e + c} = \frac{1 + 0.5}{0.1 + 0.001 + 0.5} = \frac{1.5}{0.601} = 2.5 > 1.$$

Hence, in this example, a \$1 increase in  $MB$  leads to a \$2.50 increase in  $M$ .

For the sake of comparison, let's compute the simple deposit multiplier in this example:

$$\frac{1}{r} = \frac{1}{0.1} = 10.$$

Evidently, the money multiplier  $m$  is smaller than the simple deposit multiplier  $1/r$ .

This is true in our example. And, in fact, it holds true more generally: the money multiplier  $m$  is always smaller than the simple deposit multiplier  $1/r$ .

This result follows from the fact that currency and excess reserves, which enter into our derivation of the money multiplier but not into our derivation of the simple deposit multiplier, do not participate in the process of multiple deposit creation.

### 3.1 Changes in the Required Reserve Ratio

Suppose now that  $r$ , the required reserve ratio, increases from 10% to 15%.

Let's recompute  $m$ :

$$m = \frac{1 + c}{r + e + c} = \frac{1 + 0.5}{0.15 + 0.001 + 0.5} = \frac{1.5}{0.651} = 2.3.$$

Evidently,  $r$  and  $m$  are negatively related:  $m$  falls when  $r$  rises, and  $m$  rises when  $r$  falls.

This result follows from the fact that less multiple deposit creation can occur when  $r$  rises, while more multiple deposit creation can occur when  $r$  falls.

Note that  $r$  appears in the denominator of the fraction

$$m = \frac{1 + c}{r + e + c},$$

which is another way of seeing that  $r$  and  $m$  are negatively related.

### 3.2 Changes in the Currency Ratio

Now let  $r = 0.10$  as before, but suppose that  $c$ , the currency ratio, increases from 50% to 75%:

$$m = \frac{1 + c}{r + e + c} = \frac{1 + 0.75}{0.1 + 0.001 + 0.75} = \frac{1.75}{0.851} = 2.06.$$

Evidently,  $c$  and  $m$  are also negatively related:  $m$  falls when  $c$  rises, and  $m$  rises when  $c$  falls.

This result follows from the fact that currency does not participate in the process of multiple deposit creation. Hence, when depositors want to hold more currency, less multiple deposit creation can occur, and when depositors want to hold less currency, more multiple deposit creation can occur.

Note that  $c$  appears in both the numerator and the denominator of the fraction

$$m = \frac{1 + c}{r + e + c},$$

making the inverse relationship between  $c$  and  $m$  a little more difficult to see. Nevertheless, it is possible to prove that an increase in  $c$  will always decrease  $m$ , so that  $c$  and  $m$  are always negatively related.

### 3.3 Changes in the Excess Reserve Ratio

Finally, let  $c = 0.5$  as before, but suppose that  $e$ , the excess reserve ratio, increases from 0.1% to 0.5%:

$$m = \frac{1 + c}{r + e + c} = \frac{1 + 0.5}{0.1 + 0.005 + 0.5} = \frac{1.5}{0.605} = 2.48.$$

Evidently,  $e$  and  $m$  are also negatively related:  $m$  falls when  $e$  rises, and  $m$  rises when  $e$  falls.

This result follows from the fact that excess reserves do not participate in the process of multiple deposit creation. Hence, when banks want to hold more excess reserves, less multiple deposit creation can occur, and when banks want to hold less excess reserves, more multiple deposit creation can occur.

Note that  $e$  appears in the denominator of the fraction

$$m = \frac{1 + c}{r + e + c},$$

which is another way of seeing that  $e$  and  $m$  are negatively related.

Recall, too, that for banks reserves do not pay interest while securities and loans do pay interest.

Hence, it seems reasonable to imagine that when interest rates rise, banks will find it advantageous to lower their holdings of excess reserves in order to buy more securities or make more loans.

And by when interest rates fall, banks will find it advantageous to raise their holdings of excess reserves by selling securities or making fewer loans.

Consistent with this reasoning, Mishkin's Figure 1 (p.1) displays an inverse relationship between the excess reserves ratio  $e$  and the interest rate (as measured by the federal funds rate—the interest rate on loans between banks) in the US, 1960-2002:

In the US, interest rates rose during the 1960s and 1970s, while the excess reserves ratio  $e$  fell.

Interest rates fell from 1980 through 2002, while the excess reserves ratio  $e$  rose.

Hence, we can conclude that

interest rates rise  $\Rightarrow$  excess reserves fall  $\Rightarrow m$  rises

and

interest rates fall  $\Rightarrow$  excess reserves rise  $\Rightarrow m$  falls

so that there tends to be a positive relationship between interest rates and the money multiplier.

## 4 Overview of the Money Supply Process

We now have a model that describes how changes in the monetary base  $MB$ , which the Fed can control, lead to changes in the money supply  $M$ :

$$M = m \times MB,$$

where the money multiplier

$$m = \frac{1 + c}{r + e + c}$$

depends on banks' decisions on their holdings of excess reserves as summarized by the excess reserve ratio  $e$  and depositors' decisions on their holdings of currency as summarized by the currency ratio  $c$ .

In practice, what has happened to the money supply, the monetary base, the money multiplier, and its various determinants over time?

To answer this question, consider the following table, consisting of data taken from the 2003 *Economic Report of the President* (Tables B-69, B-70, and B-71):

year	M1 (\$ billions)	MB (\$ billions)	$m$	$e$	$c$
1960	140.7	41.0	3.43	0.0067	0.257
1970	214.3	65.0	3.30	0.0015	0.295
1980	408.1	142.0	2.87	0.0018	0.399
1990	824.1	293.3	2.81	0.0029	0.432
2000	1088.8	584.0	1.86	0.0026	0.962
2002	1219.1	680.3	1.79	0.0039	1.065

Observations:

As noted earlier, the excess reserve ratio  $e$  fell as interest rates rose prior to 1980, and  $e$  rose as interest rates fell after 1980.

We know that  $e$  and  $m$  are negatively related. Hence, by themselves, these movements in  $e$  worked to increase  $m$  before 1980 and decrease  $m$  after 1980.

However, these movements in  $e$  cannot be the whole story since, in fact,  $m$  was falling even before 1980.

So we must look at least one of the other determinants of  $m$ .

The currency ratio  $c$  has risen steadily over the past 40 years.

We know that  $c$  and  $m$  are negatively related.

Hence, the long-run decline in the money multiplier  $m$  seems mainly to reflect the long-run increase in  $c$ .

Meanwhile, the M1 money supply has increased by more than 750%: from \$140.7 billion in 1960 to \$1,219.1 billion in 2002.

Since  $m$  has fallen, the increase in M1 has come about because the monetary base has increased over time.

In fact, the monetary base has increased by more than 1500%: from \$41.0 billion in 1960 to \$680.3 billion in 2002.

Since the monetary base is controlled by the Federal Reserve, we can say based on these facts that the Federal Reserve is primarily responsible for the increase in the money supply since 1960.

In this sense, the Federal Reserve is the most important player in the money supply process, even though banks, depositors, and borrowers play a role as well.